# Section 2.2
# "Digraphs & Adjacency Matrices"

Jan 2-5:37 PM

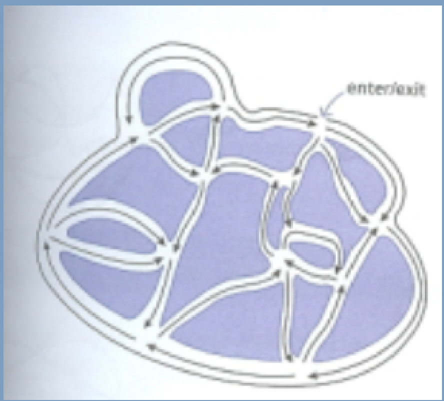
**Digraphs** = a directed graph. The edges of the graph can be traveled only in the indicated direction.

They are used to represent networks in which direction is important.

Example: Hockey drill paths, flight paths, etc...

**NETWORK**


**DIGRAPH**

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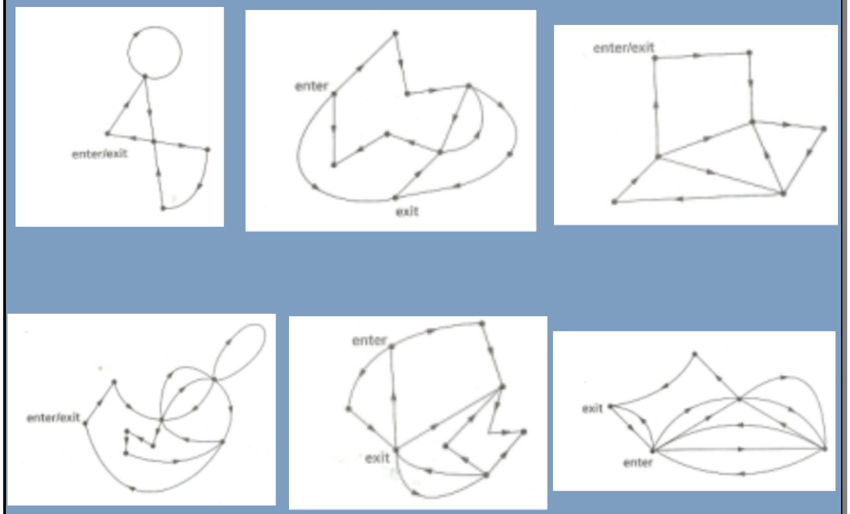
# Investigation 2

A. For each maze, decide whether all paths can be traveled without repeating a path. If it can not, draw a circle where the issue is....




B. If the maze cannot be traveled without repeating a path, draw extra paths to solve the problem.

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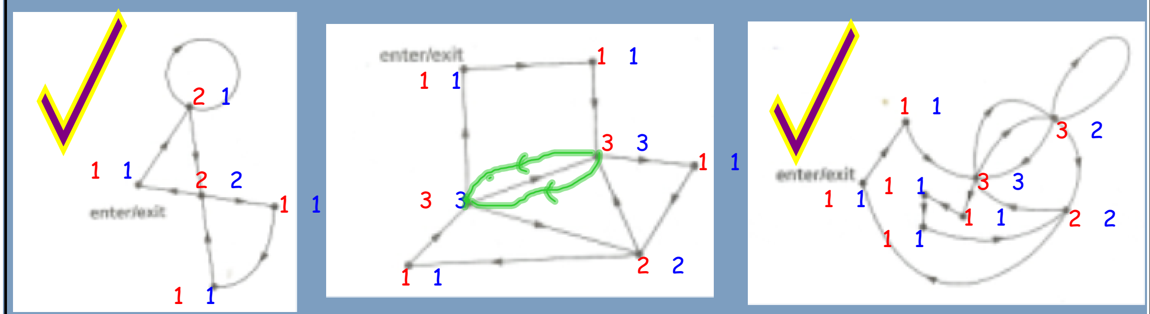
# Investigation Questions:

1. Name the vertices of the digraphs that have the entrance and exit at the same vertex. At each vertex, record the number of paths leading in, the number leading out, and the total number. What do you notice?

Incoming Outgoing




What do you notice? Digraphs with an entrance and exit at the same vertex have an even number of paths connected to each vertex. For each vertex, the number of paths leading in and the number of paths leading out are equal. The only exception is if there is a loop.

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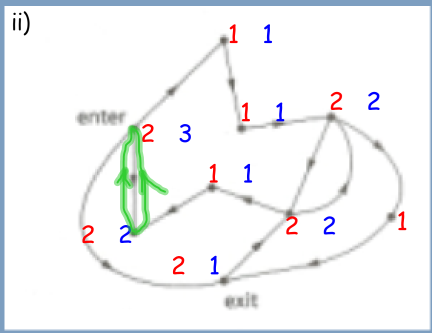
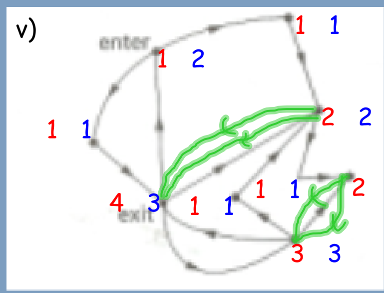
2. Look at the graphs that have the entrance and exit at different vertices. Name the vertices and record the number of paths leading in, then number leading out and the total number.

Incoming Outgoing




What do you notice? Digraphs with the entrance and exit at different vertices have an odd number of paths only at the entrance and exit. For each of the other vertices, the number of paths leading in and out are be equal.

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# Key Notes:

**A digraph can be traveled without repeating if:**

**Different entrance and exit --> there can be two odd vertices in a digraph as long as one is the entrance and one the exit**

**Same entrance and exit --> all vertices must be even with the same number of paths entering as exiting. Loops do not count.**

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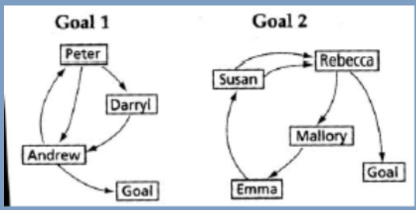
# Question 3, page 65

Each network diagram shows the passes that led to a goal during a hockey game.




a) Re-draw each network as a digraph.


b) What do the vertices and paths represent?

The vertices represent players and the edges represent passes.

c) For goal 1 who passed first & who scored?

Peter passed first, Andrew scored.

d) For goal 2 who passed first & who scored?

Susan passed first, Rebecca scored.

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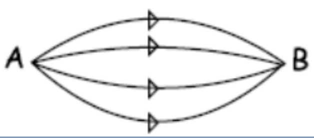
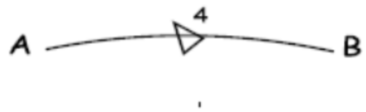
# Focus A: Simplifying Digraphs

## Simplification Rules:

1. Digraphs with many directed edges between adjacent vertices can be simplified. Use numbers to represent the number of edges going in the same direction between two vertices.

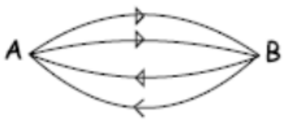
For example:

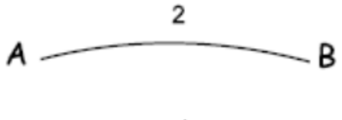



2. If there is no direction indicated on an edge, this means that there is an equal number of edges going both ways.

For example:

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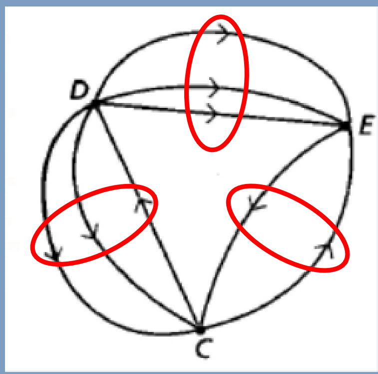
A graph becomes complicated when there are many directed paths. There are ways to simplify the graph and display the same information.

Example:

The digraph represents the network of direct flights among three airports: Charlottetown (C), Deer Lake (D), and Edmunston (E). Simplify as much as possible.

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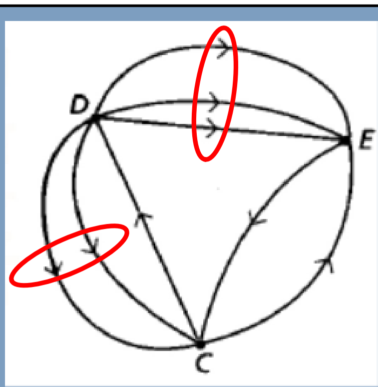
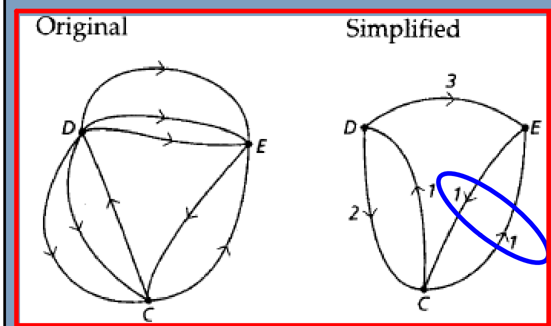
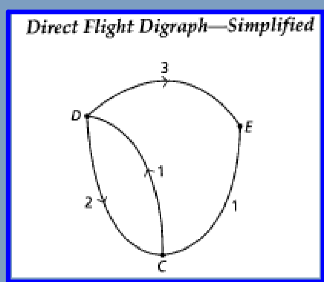



Simplify Common Paths




Simplify Same Number, Opposite Direction

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Question 4:

a) Draw a graph to show two flights from E to D

b) Suppose three flights from E to D were added to the original. How would this change the simplified graph?

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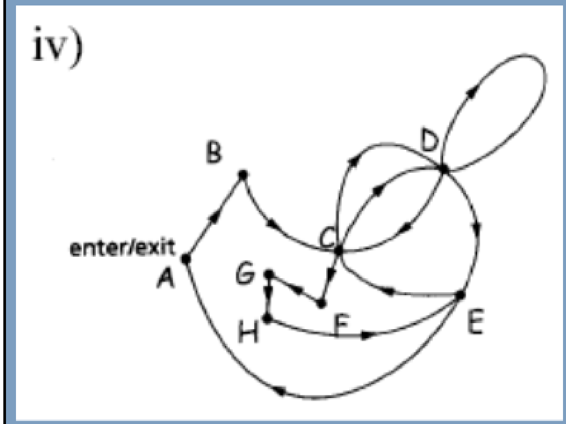
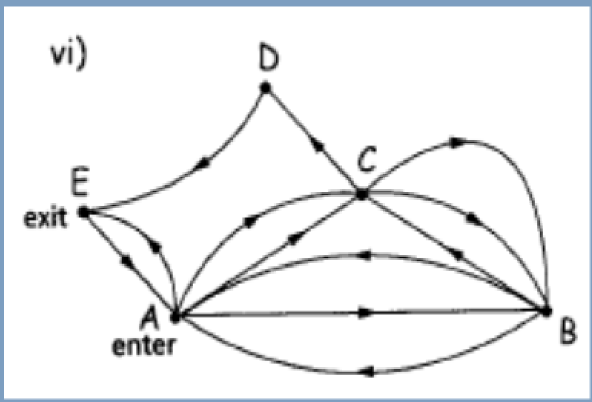
## Simplify the following Digraphs

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# Class work / Homework

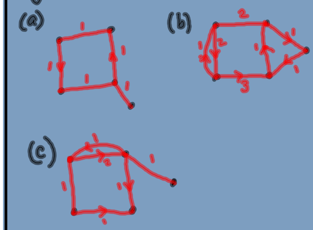
- Page 67 # 5

## Review of Networks and Digraphs:

- Page 92 #'s 1 (a-d), 2, & 5
- Page 93 #'s 6 & 7

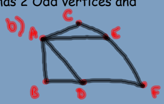
Jan 4-8:21 AM

Pg. 67 #5



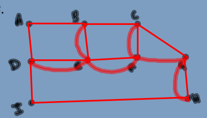
Pg. 92 + 93

#1
a) It is a network because it only has 2 Odd vertices and therefore can be travelled.
c) Vertices represent friends.
d) Edges represent conversations.



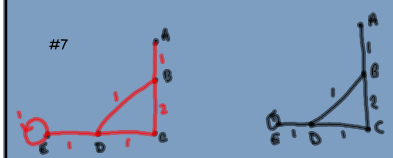
#2
Trail 1 & 3: Yes, it works because all vertices are even.
Trail 2: Yes, it works because there are only 2 odd vertices.

#3: It doesn't work because there are too many odd vertices.



#6
a) The snow plow goes down one side and comes back on the other side.
b) Yes it can work. Route: ABDBCBCDEDCBA

#7



Jan 7-8:12 AM

# Networks & Digraphs Quiz Review

**Terminology:** Network, Network Graph, Vertex, Edge, Even Vertex, Odd Vertex, digraph, path

**Concepts:**
- Decide if a network can be completed
- Draw a network from a diagram
- Decide if a digraph can be completed
- Fix digraphs that can not be completed
- Simplify a digraph from a diagram

**Types of Questions:**
Page 57 part A
Page 59 #5
Page 64 part A
Page 93 #6 and 7

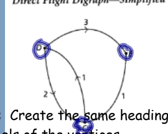
Jan 4-8:33 AM

## Focus B: Representing Digraphs as Adjacency Matrices

Steps to go from a Digraph to a Table:

We will use the simplified flight digraph from Investigation A

Direct Flight Digraph—Simplified




Step 1: Create the same headings for both rows and columns using the labels of the vertices.

Step 2: Fill in the table by entering the direct flights from each airport in each row to each airport in each column.

| | | TO | | |
|---|---|---|---|---|
| | | C | D | E |
| FROM | C | 0 | 1 | 1 |
| | D | 2 | 0 | 3 |
| | E | 1 | 0 | 0 |

Steps to go from a Table to Adjacency Matrix:

A matrix is a rectangular array of numbers (plural is matrices).

The matrix form for the direct flight table is:

$$\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 3 \\ 1 & 0 & 0 \end{pmatrix}$$

- Element: Each number in a matrix.
  - The above matrix has nine elements.
- Dimensions: The size of a matrix, described as the # of rows by the # of columns.
  - The above matrix is a 3 by 3 matrix
- Adjacency Matrix: a special matrix that organizes information about connections between adjacent vertices in a network.
- Matrices are actually tables without headings

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## Practice Questions:

Pg. 68 #7-11

→use the digraph on pg.67

→ use the matrix on pg.68

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### Questions Pg. 68 #7-11: ANSWERS

7. a) 3

b) From the digraph, you count the number of edges from Deer Lake to Charlottetown (2), and the number from Charlottetown to Deer Lake (1), and add them.

c) From the matrix, you add the element in row 2, column 1 (2) to the element in row 1 column 2 (1).

8. a) Greatest Depatures: Deer Lake

Greatest Arrivals: Edmunston

b) For departures, you look for the vertex on the digraph with the greatest number of edges leading away from it; for arrivals, look for the greatest number of edges leading into a vertex.

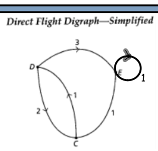
c) To find the greatest number of departures using the matrix, find the sum of each row and then look for the row with the greatest sum; for arrivals, find the sum of each column and the look for the column with the greatest sum.

9. a) You can fly to Deer Lake through Charlottetown.

b) It is zero because you can't fly directly from Edmunston to Deer Lake.

10. There are zeros along the diagonal because there are no flights that depart from and arrive at the same airport (with no stops along the way).

11. a)




b)

$$\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 3 \\ 1 & 0 & 1 \end{pmatrix}$$

c) There is a 1 on the diagonal now to show that there is a flight that takes off from and arrives at the same airport without stopping anywhere along the way.

Jan 5-6:43 PM